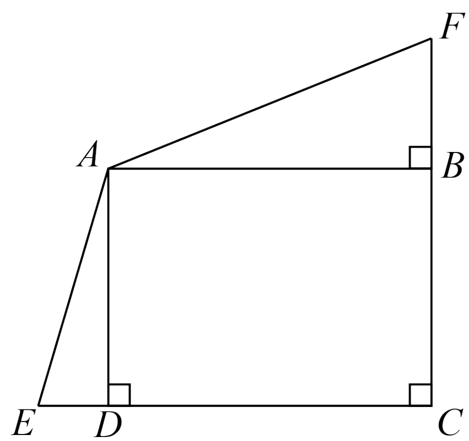
Problem of the Week
Problem C and Solution
Around the Outside

Problem

Two line segments, CE and CF, are perpendicular to each other, each with length 10. Rectangle $ABCD$ is drawn so that D is on CE, B is on CF with $BF = 4$, and the diagonal of $ABCD$ has length 10. Line segments EA and AF are then drawn. Determine the perimeter of quadrilateral $AFCE$, rounded to one decimal place.



NOTE: You may find the following useful:

The *Pythagorean Theorem* states, "In a right-angled triangle, the square of the length of hypotenuse (the side opposite the right angle) equals the sum of the squares of the lengths of the other two sides."

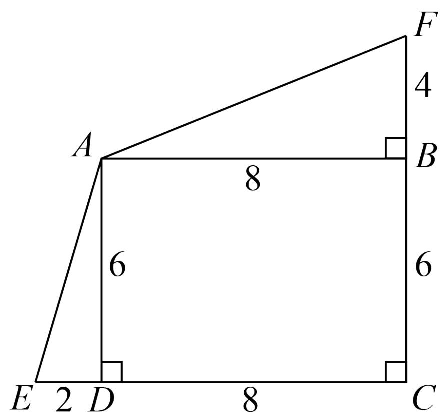
For example, if c is the length of the hypotenuse, and a and b are the lengths of the other two sides, then $c^2 = a^2 + b^2$.

Solution

First, $BC = CF - BF = 10 - 4 = 6$. Since $ABCD$ is a rectangle, $AD = BC = 6$. We then use the Pythagorean Theorem in $\triangle ABC$.

$$\begin{aligned} AB^2 &= AC^2 - BC^2 \\ &= 10^2 - 6^2 \\ &= 100 - 36 \\ &= 64 \end{aligned}$$

Therefore $AB = 8$, since $AB > 0$. Since $ABCD$ is a rectangle, $CD = AB = 8$. Then $DE = CE - CD = 10 - 8 = 2$. These lengths are shown on the diagram.



We then use the Pythagorean Theorem in $\triangle ADE$.

$$\begin{aligned} AE^2 &= AD^2 + DE^2 \\ &= 6^2 + 2^2 \\ &= 36 + 4 \\ &= 40 \end{aligned}$$

Therefore $AE = \sqrt{40}$, since $AE > 0$.

We then use the Pythagorean Theorem in $\triangle ABF$.

$$\begin{aligned} AF^2 &= AB^2 + BF^2 \\ &= 8^2 + 4^2 \\ &= 64 + 16 \\ &= 80 \end{aligned}$$

Therefore $AF = \sqrt{80}$, since $AF > 0$.

Thus, the perimeter of $AFCE$ is equal to
$AF + CF + CE + AE = \sqrt{80} + 10 + 10 + \sqrt{40} \approx 35.3$.