



Problem of the Week

Problem C and Solution

All the Digits

Problem

Georgina listed the integers from 1 to 13:

$$1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13$$

She determined that the sum of all of the digits of the integers in this list is

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + (1 + 0) + (1 + 1) + (1 + 2) + (1 + 3) = 55$$

She then challenged you to determine the sum of all of the digits of the integers from 1 to 100. What sum would you get?

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Solution

In the integers from 1 to 100, each digit from 0 to 9 appears as the units digit of a number exactly ten times. For example, the digit 1 appears as the units digit in the numbers 1, 11, 21, 31, 41, 51, 61, 71, 81, 91, a total of 10 numbers. Therefore, the sum of all of the units digits is

$$\begin{aligned} &10(1) + 10(2) + 10(3) + 10(4) + 10(5) + 10(6) + 10(7) + 10(8) + 10(9) + 10(0) \\ &= 10(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 0) \\ &= 10(45) \\ &= 450 \end{aligned}$$

Similarly, each digit from 1 to 9 appears as the tens digit of a number exactly ten times. For example, the digit 1 appears as the tens digit in the numbers 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, a total of 10 numbers. The digit 0 appears as the tens digit of a number once. Therefore, the sum of all of the tens digits is

$$\begin{aligned} &10(1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9) + 0 = 10(45) \\ &= 450 \end{aligned}$$

The number 100 is the only number with a hundreds digit. We need to add 1 to our final sum.

Therefore, the sum of all of the digits of the integers from 1 to 100 is $450 + 450 + 1 = 901$.